

MA114 Summer II 2018
Worksheet 3b – Trig Integrals
6/12/18

1. Compute the following integrals.

$$\begin{aligned}
 \text{a) } \int \tan^5(x) \sec^6(x) dx &= \int \tan^4(x) \sec^5(x) \sec(x) \tan(x) dx \\
 &= \int (\sec^2(x) - 1)^2 \sec^5(x) \sec(x) \tan(x) dx && u = \sec(x) \\
 & && du = \sec(x) \tan(x) \\
 &= \int (u^2 - 1)^2 u^5 du \\
 &= \int (u^4 - 2u^2 + 1) u^5 du \\
 &= \int u^9 - 2u^7 + u^5 du = \boxed{\frac{1}{10} \sec^{10}(x) - \frac{2}{8} \sec^8(x) + \frac{1}{6} \sec^6(x) + C}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } \int \csc^2(3-2x) dx &= -\frac{1}{2} \int \csc^2(u) du \\
 u &= 3-2x \\
 du &= -2 dx \\
 &= \frac{1}{2} \cot(u) + C \\
 &= \boxed{\frac{1}{2} \cot(3-2x) + C}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } \int \cot^5(x) \csc^5(x) dx &= \int \cot^4(x) \csc^4(x) \cot(x) \csc(x) dx \\
 &= \int (\csc^2(x) - 1)^2 \csc^4(x) \cot(x) \csc(x) dx && u = \csc(x) \\
 & && du = -\csc(x) \cot(x) dx \\
 &= \int (u^2 - 1)^2 u^4 (-1) du \\
 &= \int -u^8 + 2u^6 - u^4 du \\
 &= \boxed{-\frac{1}{9} \csc^9(x) + \frac{2}{7} \csc^7(x) - \frac{1}{5} \csc^5(x) + C}
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } \int \tan^3(t) dt &= \int (\sec^2(t) - 1) \tan(t) dt \\
 &= \int \sec^2(t) \tan(t) dt - \int \tan(t) dt \\
 &= \boxed{\frac{1}{2} \tan^2(t) - \ln|\sec(t)| + C} \\
 &\quad \uparrow \\
 &\quad \text{u-sub for } \tan(t)
 \end{aligned}$$