

# Worksheet 26 Solutions

1) a)  $y = \sin(3x) + 2e^{4x}$   
 $y' = 3\cos(3x) + 8e^{4x}$   
 $y'' = -9\sin(3x) + 32e^{4x}$

$$y'' + 9y = 50e^{4x}$$

$$-9\sin(3x) + 32e^{4x} + 9(\sin(3x) + 2e^{4x}) = 50e^{4x}$$

$$32e^{4x} + 18e^{4x} = 50e^{4x}$$

$$50e^{4x} = 50e^{4x} \quad \checkmark$$

The function is a solution to the DE

2)  $y = e^{\alpha x}$   
 $y' = \alpha e^{\alpha x}$   
 $y'' = \alpha^2 e^{\alpha x}$

$$y'' + y' - 12y = 0$$

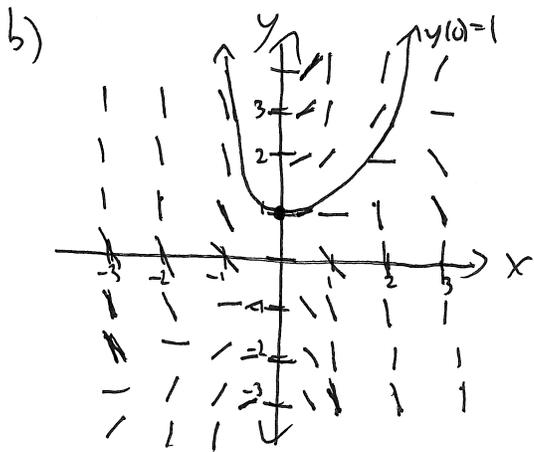
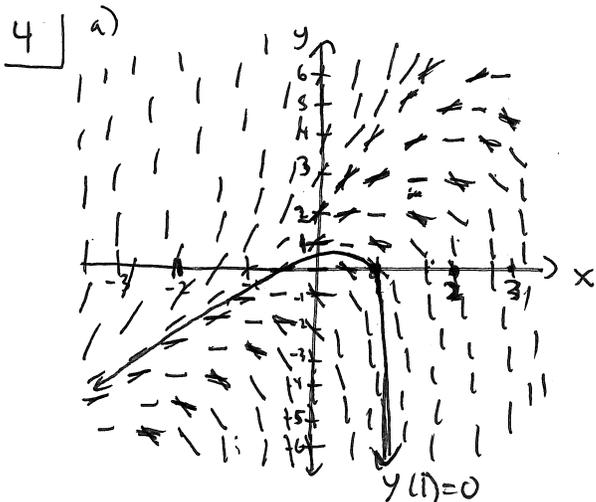
$$\alpha^2 e^{\alpha x} + \alpha e^{\alpha x} - 12e^{\alpha x} = 0$$

$$e^{\alpha x}(\alpha^2 + \alpha - 12) = 0$$

$$\alpha^2 + \alpha - 12 = 0$$

$$(\alpha + 4)(\alpha - 3) = 0$$

$$\alpha = -4, \alpha = 3$$



b)  $\frac{dy}{dx} > 0$  for any value of  $y$  b/c  $y^2 + 6 > 0$ .

So the function is increasing everywhere.

c) A differential equation is linear if every term involving  $y$  or  $y^{(k)}$  is a linear function.

3)  $y' = 2x - \frac{2c}{x^3}$

$$x y' + 2y = 4x^2$$

$$x(2x - \frac{2c}{x^3}) + 2(x^2 + \frac{c}{x^2}) = 4x^2$$

$$2x^2 - \frac{2c}{x^2} + 2x^2 + \frac{2c}{x^2} = 4x^2$$

$$4x^2 = 4x^2 \quad \checkmark$$

$$y(6) = 5: \quad 5 = 6^2 + \frac{c}{6^2}$$

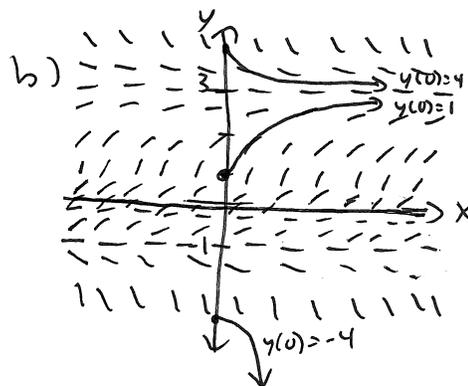
$$-31 = \frac{c}{36}$$

$$c = (-31)(36) = -1116$$

5) a)  $y' = y^2(3-y)(y+1)$

$$0 = y^2(3-y)(y+1)$$

$$y = 0, y = 3, y = -1$$



c)  $y = 0$ : semi-stable

$y = 3$ : stable

$y = -1$ : unstable

6) a)  $\lim_{t \rightarrow \infty} y(t) = 3$  if  $y(0) = 1$     c)  $\lim_{t \rightarrow \infty} y(t) = -\infty$  if  $y(0) = -4$

b)  $\lim_{t \rightarrow \infty} y(t) = 3$  if  $y(0) = 4$