MA 114 MathExcel Worksheet G: Exam 02 Review

1. It is easy to see that $\lim_{n\to\infty} \sin\left(\frac{1}{n}\right) = 0$. What does this say about the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$? Does the series converge or diverge? How can you tell?

2. Determine whether each sequence converges, and if so, to what value.

   (a) $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ (b) $\left\{2^{n+1} - 2^n\right\}_{n=2}^{\infty}$ (c) $\left\{\frac{1}{n} - \frac{1}{n+1}\right\}_{n=1}^{\infty}$ (d) $\left\{1 + \frac{(-1)^n}{2^n}\right\}_{n=5}^{\infty}$

3. Determine whether each series converges, and if so, to what value.

   (a) $\sum_{n=0}^{\infty} \frac{2^n - 5^n}{8^n}$ (b) $\sum_{k=1}^{\infty} \left(\frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+1}}\right)$ (c) $\sum_{m=1}^{\infty} (\ln(m) - \ln(m+1))$

4. Show that the power series $f(x), g(x)$, and $h(x)$ below have the same radius of convergence. Then show that $f(x)$ diverges at both endpoints, $g(x)$ converges at one endpoint but diverges at the other, and $h(x)$ converges at both.

   $$f(x) = \sum_{n=1}^{\infty} \frac{x^n}{3^n} \qquad g(x) = \sum_{n=1}^{\infty} \frac{x^n}{n\,3^n} \qquad h(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^2\,3^n}$$

5. Determine if each of the following series converges or diverges. For series that have some negative terms, specify whether the convergence is absolute or conditional.

   (a) $\sum_{j=0}^{\infty} \frac{2^{2j}}{j!}$

   (b) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n + \ln(n)}}$

   (c) $\sum_{n=2}^{\infty} \frac{n}{\sqrt{n^5 + 5}}$

   (d) $\sum_{n=1}^{\infty} \frac{n^2}{(n^3+1)^{1.01}}$

   (e) $\sum_{n=1}^{\infty} \left(\frac{3}{4n}\right)^n$

   (f) $\sum_{n=1}^{\infty} \frac{\pi^{7n}}{e^{8n}}$

   (g) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n} + \sqrt{n+1}}$

   (h) $\sum_{k=1}^{\infty} \frac{1}{k + \sqrt{k}}$

   (i) $\sum_{n=2}^{\infty} \frac{1}{n^{\ln(n)}}$

   (j) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$

6. Find the interval of convergence of each of the following power series. Remember to check endpoints!

(a) $\sum_{n=0}^{\infty} \frac{n^6}{n^8+1}(x-3)^n$ (b) $\sum_{n=0}^{\infty}(nx)^n$ (c) $\sum_{n=2}^{\infty} \frac{(x+4)^n}{(n\ln(n))^2}$

7. Recall that $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$ and $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$.

   (a) Find the Maclaurin series representations of $\sinh(x)$ and $\cosh(x)$.

   (b) What are the intervals of convergence of the series you found in part (a)?

   (c) Use the Maclaurin series to verify that $\frac{d}{dx}(\sinh(x)) = \cosh(x)$ and $\frac{d}{dx}(\cosh(x)) = \sinh(x)$.