MathExcel Worksheet C: Exam I Review

1. Evaluate the following integrals:

   (a) $\int \cos^5(x)\sin^2(x)dx$

   (b) $\int \frac{1}{\sqrt{4-x^2}}dx$

   (c) $\int \frac{-2x}{\sqrt{4-x^2}}dx$

   (d) $\int_{\pi/2}^{\pi} \sin^2(x)\sin(2x)\cos(x)dx$

   (e) $\int_1^{\sqrt{3}} \arctan(1/x)dx$ (Hint: integrate by parts with $u = \arctan(1/x)$ and $dv = dx$)

   (f) $\int \tan^3(x)\sec^2(x)dx$

2. Evaluate $\int_0^{\pi} \sin^2(mx)dx$ for m any nonzero integer.

3. Evaluate the integral using integration by parts as a first step

$$\int \frac{\arcsin(x)}{x^2}dx$$

4. Solve the integral $\int \frac{dx}{x^2-1}$ in the following ways and verify that the answers agree.

   a.) trigonometric substitution

   b.) use the partial fraction decomposition

$$\frac{1}{x^2-1} = \frac{1/2}{x-1} - \frac{1/2}{x+1}$$

5. Find the partial fraction decomposition of the following rational functions. Do **NOT** evaluate any integrals (unless you really want to...).

   (a) $\frac{x^2+4x+12}{(x+2)(x^2+4)}$

   (b) $\frac{x^2-4x+8}{(x-1)^2(x-2)^2}$

6. A function f is know to have a fourth derivative with the property that $|f^{(4)}(x)| \leq 6$ on $[-1,5]$. Determine how many subintervals are required so that the Simpson rule used to approximate $\int_{-1}^{5} f(x)\ dx$ incurs an error less than .0001.

7. A table of values for a continuous function f is shown below. If four equal subintervals of $[0,2]$ are used, what is the Simpsons rule approximation for $\int_0^2 f(x)\ dx$?

x	0.0	0.5	1.0	1.5	2.0
f(x)	2	8	6	12	10

8. For each of the following integrals, decide which is improper. For the improper integrals, set up **BUT DO NOT EVALUATE** the corresponding limit problem.

(a) $\displaystyle\int_{-\infty}^{3} x^2\ dx$

(b) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2}\ dx$

(c) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \tan\theta\ d\theta$

(d) $\displaystyle\int_{0}^{10000} \ln(x^2+1)\ dx$

9. For the integrals below, determine if the integral is convergent or divergent. Evaluate the convergent integrals.

(a) $\displaystyle\int_{1}^{\infty} \frac{1}{x^{3/2}}\ dx$

(b) $\displaystyle\int_{-\infty}^{\infty} xe^{-x^2}\ dx$

(c) $\displaystyle\int_{4}^{\infty} \frac{1}{x}\ dx$

(d) $\displaystyle\int_{1}^{\infty} \frac{1+e^{-x}}{\sqrt{x}}\ dx$ (Hint: see whether $\displaystyle\int_{1}^{\infty} \frac{1}{\sqrt{x}}\ dx$ is convergent or divergent)

(e) $\displaystyle\int_{-1}^{0} \frac{e^{1/x}}{x^3}\ dx$

(f) $\displaystyle\int_{0}^{2} \frac{x}{x-1}\ dx$