MA 114 Worksheet #09: Recursive Sequences and Series

1. Write out the first five terms of
   (a) $a_0 = 0$, $a_1 = 1$ and $a_{n+1} = 3a_{n-1} + {a_n}^2$.
   (b) $a_1 = 6$, $a_{n+1} = \dfrac{a_n}{n}$.
   (c) $a_1 = 2$, $a_{n+1} = \dfrac{a_n}{a_n + 1}$.
   (d) $a_1 = 1$, $a_{n+1} = \sqrt{\left(\dfrac{2}{a_n}\right)^2 + 1}$.
   (e) $a_1 = 2$, $a_2 = 1$, and $a_{n+1} = a_n - a_{n-1}$.
2. (a) For what values of x does the sequence $\{x^n\}_{n=1}^{\infty}$ converge?
   (b) For what values of x does the sequence $\{n^x\}_{n=1}^{\infty}$ converge?
   (c) If $\lim\limits_{n\to\infty} b_n = \sqrt{2}$, find $\lim\limits_{n\to\infty} b_{n-3}$.
3. (a) Determine whether the sequence defined as follows is convergent or divergent:

$$a_1 = 1 \qquad a_{n+1} = 4 - a_n \qquad \text{for } n > 1.$$

   (b) What happens if the first term is $a_1 = 2$?
4. A fish farmer has 5000 catfish in his pond. The number of catfish increases by 8% per month and the farmer harvests 300 catfish per month.
   (a) Show that the catfish population P_n after n months is given recursively by

$$P_n = 1.08P_{n-1} - 300 \qquad P_0 = 5000.$$

   (b) How many catfish are in the pond after six months?

Math Excel Worksheet #09: Recursive Sequences

1. The Fibonacci numbers $\{F_n\}_{n=0}^{\infty}$ are defined recursively as follows:

$$F_0 = 1, F_1 = 1, \text{ and for } n \geq 2,\ F_n = F_{n-1} + F_{n-2}.$$

Fill out the following table.

n	0	1	2	3	4	5	6	7	8	9	10
F_n	1	1									
F_{n+1}/F_n											

The sequence F_{n+1}/F_n converges to the golden ratio, $\phi = \frac{1+\sqrt{5}}{2}$, studied by the Ancient Greeks.

2. A bee keeper has 400 bees in each hive. The number of bees increases by 1.2% per month and the bee keeper harvests honey each month resulting in the death of 20 bees in each hive each month.

   (a) Find a recursive formula for the bee population after n months for a single hive.

   (b) How many bees are there after six months if there are 6 hives?

   (c) Is the bee population increasing or decreasing? If it is decreasing, when will the bees all die out according to your model?

3. Let $\{a_n\}_{n=1}^{\infty}$ be defined recursively by $a_1 = 5$, and for $n \geq 2$, $a_n = a_{n-1}/3$. Find a non-recursive formula for a_n.