Integration Strategy for MA 114

Here are some general ideas to think about when you are trying to get started on an integration problem. I strongly suggest that before the first exam you go through the attached list of integrals and decide what method of integration you would use for each, without actually performing the integration.

1. Is the integral an improper integral? If it is, rewrite it in terms of a limit of a proper integral and continue.

2. Do you already know an antiderivative for this function? (e.g. polynomials, $\sin(x)$, e^x, $\frac{1}{x}$, $\frac{1}{1+x^2}$, $\sec(x)$, etc.) If you do, you're done!

3. Are there any simplifications you can make? (e.g $\sqrt{x}(x^2 - x + 1) = x^{5/2} - x^{3/2} + x^{1/2}$)

4. Look for a good substitution. Can you find a function $u = f(x)$ so that $du = f'(x)dx$ appears in your integrand and you can simplify the expression somehow by making the substitution?

5. Is the integrand a rational function? If it is, set up a partial fraction decomposition.

6. Is the integrand composed of trig functions? Try to use some trig identities to set up a good substitution.

7. Does the integrand contain any of the following?

$$(a^2 + x^2), \qquad (a^2 - x^2), \qquad (x^2 - a^2)$$

   First double check if you can make a u-substitution work. If not, draw a triangle and make a trig substitution.

8. Is the integrand a product of two functions (remember that 1 is a function too!)? If it is, try doing integration by parts. Remember that your goal is that u becomes simpler when you differentiate it and dv doesn't get worse when you integrate it. Also, you need to know an antiderivative for dv! Watch out for problems where your integral returns to the original integral after a few uses of integration by parts.

9. Repeat 3-8 until everything falls into category 2.

10. Here are some integrals which use all of the techniques we have learned so far. For each one, decide what technique seems best to you for the problem.

(a) $\displaystyle\int_0^1 (3y+1)^{\sqrt{2}}\,dy$

(b) $\displaystyle\int \frac{\sin^3(x)}{\cos(x)}\,dx$

(c) $\displaystyle\int x\sin(x)\cos(x)\,dx$

(d) $\displaystyle\int \frac{1}{t^3\sqrt{t^2-1}}\,dt$

(e) $\displaystyle\int \frac{2u-3}{u^3+3u}\,du$

(f) $\displaystyle\int e^2\,dz$

(g) $\displaystyle\int_e^\infty \frac{1}{(y-2)^{3/2}}\,dy$

(h) $\displaystyle\int \frac{\cos(1/x)}{x^3}\,dx$

(i) $\displaystyle\int \frac{1+\sin(x)}{1+\cos(x)}\,dx$

(j) $\displaystyle\int \frac{4^x+10^x}{2^x}\,dx$

(k) $\displaystyle\int \frac{dx}{\sqrt{x}+x\sqrt{x}}$

(l) $\displaystyle\int \frac{e^{2\theta}}{1+e^\theta}\,d\theta$

(m) $\displaystyle\int \frac{x^2}{x^6+3x^3+2}\,dx$

(n) $\displaystyle\int \frac{dt}{1+\cos(t)}$

(o) $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

(p) $\displaystyle\int_1^3 \frac{e^{3/x}}{x^2}\,dx$

(q) $\displaystyle\int \theta\tan^2(\theta)\,d\theta$

(r) $\displaystyle\int x\sin^2(x)\cos(x)\,dx$