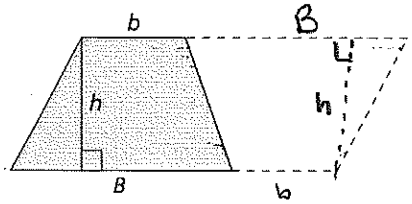
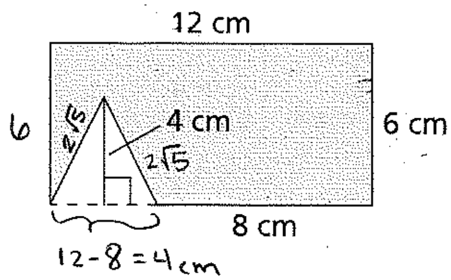
14. Explain how you can derive the formula for the area of a trapezoid based on the area of a parallelogram.



• MAKE AN EXACT COPY OF THE TRAPEZOID, FLIP IT UPSIDE DOWN AND LINE IT UP WITH THE ORIGINAL TRAPEZOID. THE NEW FIGURE HAS EXACTLY TWICE THE AREA OF THE TRAPEZOID AND IS A PARALLELOGRAM WITH LENGTH b+B AND HEIGHT h.

HENCE $2 \cdot A_{trap} = A_{new} = (b+B)h \Rightarrow \boxed{A_{trap} = \frac{1}{2}(b+B)h}$.

15. (a) Find the area of the figure.

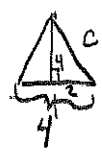


$A = A_{retangle} - A_{triangle}$

$= (12)(6) - \frac{1}{2}(4)(4)$

$= 48 - 8$

$= \boxed{40 cm^2}$



$c^2 = 4^2 + 2^2 = 16 + 4 = 20$

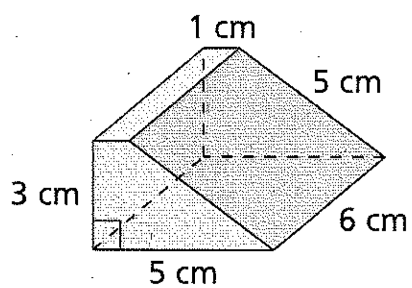
$\Rightarrow c = \sqrt{20} = 2\sqrt{5}$

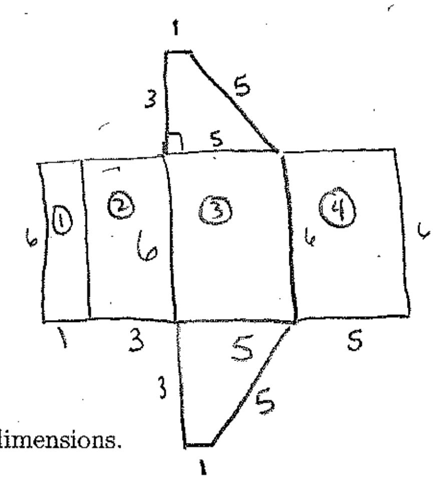
(b) Assuming the triangle is isosceles, find the perimeter of the figure.

$P = 12 + 6 + 8 + 2\sqrt{5} + 2\sqrt{5} + 6$

$= \boxed{32 + 4\sqrt{5} \text{ cm}}$

16. You are given the following figure:





(a) Sketch the net of the figure. Make sure to label all dimensions.

(b) Find the surface area of the figure.

$SA = 2A_{trap} + A_1 + A_2 + A_3 + A_4 = 2\left[\frac{1}{2}(1+5)(3)\right] + (5 \cdot 6) + (5 \cdot 6) + (3 \cdot 6) + (1 \cdot 1)$

$= (6)(3) + 30 + 30 + 18 + 1$

$= 18 + 60 + 18 + 1$

$= 37 + 60 = \boxed{97 \text{ cm}^2}$

(c) Find the volume of the figure.

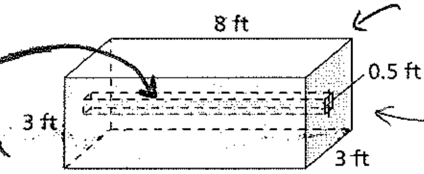
$V = A_{base} \cdot h$ (trapezoid; 6)

$= \left[\frac{1}{2}(5+1) \cdot 3\right] \cdot 6$

$= \frac{1}{2}(6) \cdot 3 \cdot 6$

$= 3 \cdot 3 \cdot 6 = \boxed{54 \text{ cm}^3}$

17. A smaller rectangular prism is cut out of a larger rectangular prism as shown:



4 sides all with same area

$\leftarrow A_{top} = A_{bottom}$

$\leftarrow A_{left} = A_{right}$

(a) What is the surface area of the original prism? What is the surface area of the new solid?

$A_{large} = 2A_{top} + 2A_{left} = 2(8\cdot 3) + 2(3\cdot 3 - 0.5\cdot 0.5)$

$= 48 + 2(9 - 0.25) = 48 + 18 - 0.5 = 65.5\ ft^2$

$A_{small} = 4(0.5\cdot 8) = 16$

$SA = 65.5 + 16 = \boxed{81.5\ ft^2}$

(b) What is the volume of the original prism? What is the volume of the new solid?

$V_{large} = l\cdot w\cdot h = (3)(3)(8) = 72\ ft^3$

$V_{small} = l_{small}\cdot w_{small}\cdot h_{small} = (0.5)(0.5)(8) = 2\ ft^3$

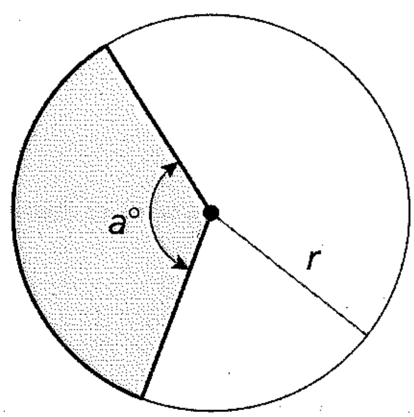
$V = V_{large} - V_{small} = 72 - 2 = \boxed{70\ ft^3}$

18. Explain how to change the side lengths of the base of a square pyramid so that the volume of the square pyramid doubles.

$V_{pyr} = \frac{1}{3}B\cdot h = \frac{1}{3}s^2h$

$2V_{pyr} = 2\left(\frac{1}{3}s^2h\right) = \frac{1}{3}(2s^2)h = \frac{1}{3}(\sqrt{2}\,s)^2h \Rightarrow$ MULTIPLY EACH SIDE BY $\sqrt{2}$.

19. Give the formula for the area A and arc length L of the sector of the circle with central angle $a°$:

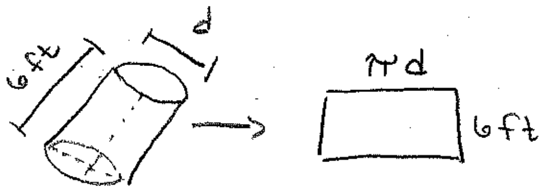


area of circle $\rightarrow$ $\frac{A}{\pi r^2} = \frac{a}{360}$

$\boxed{A = \frac{a}{360}\cdot \pi r^2}$

circumference of circle $\rightarrow$ $\frac{L}{2\pi r} = \frac{a}{360}$

$\boxed{L = \frac{a}{360}\cdot 2\pi r}$



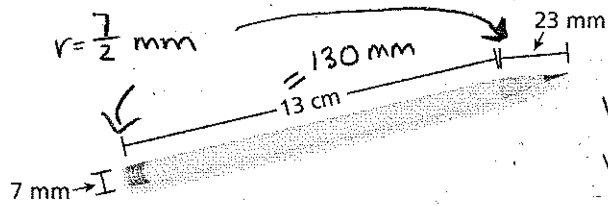
20. A log that is 6 feet long is rotated on a lathe to slice off a thin, rectangular sheet of wood called a veneer. Compare the areas of the veneer produced from one rotation of the log when its diameter is $d = 8$ inches and when its diameter is $d = 4$ inches.

$$A_8 = \pi(8)\cdot 6 = 2\cdot\pi(4)\cdot 6 = 2A_4$$
$$A_4 = \pi(4)\cdot 6$$

SO A_8 (diameter = 8) IS TWICE THE AREA OF A_4 (diameter = 4).

21. The eraser of the pencil shown is a hemisphere. The sharpened end of the pencil is a cone. Find the total volume of the pencil and its eraser in cubic centimeters.

$r = \frac{7}{2}$ mm



$$V = V_{eraser} + V_{body} + V_{end}$$
$$V_{eraser} = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi\left(\frac{7}{2}\right)^3 = \frac{7^3}{12}\pi$$
$$V_{body} = \pi r^2\cdot h = \pi\left(\frac{7}{2}\right)^2(130) = \frac{7^2\cdot 130}{4}\pi$$
$$V_{end} = \frac{1}{3}\pi r^2\cdot h = \frac{1}{3}\pi\left(\frac{7}{2}\right)^2\cdot(23) = \frac{7^2\cdot 23}{12}\pi$$
$$V = \frac{7^3}{12}\pi + \frac{7^2\cdot 130}{4}\pi + \frac{7^2\cdot 23}{12}\pi = \frac{7^2\pi}{12}(7 + 390 + 23) = \frac{7^2\pi}{12}\cdot 420 = \boxed{35\cdot 7^3\pi \text{ mm}^3}$$

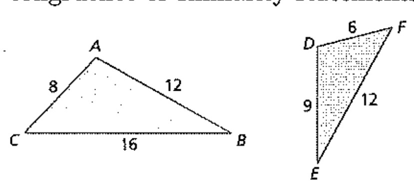
22. State the triangle congruence and similarity theorems.

congruent angles AND sides.
- SAS congruence
- ASA congruence
- SSS congruence

- SAS similarity
- AA similarity
- SSS SIMILARITY

congruent angles, proportional sides

23. Determine whether the two triangles are congruent or similar. Explain your reasoning. Write the congruence or similarity statements if possible.



NOTE THAT
$$\frac{CB}{FE} = \frac{16}{12} = \frac{4}{3},\quad \frac{AB}{DE} = \frac{12}{9} = \frac{4}{3},\quad \frac{AC}{DF} = \frac{8}{6} = \frac{4}{3}$$
. SO ALL PAIRS OF SIDES ARE PROPORTIONAL AND HENCE $\triangle ABC \sim \triangle DEF$ BY SSS SIMILARITY.

24. Consider the triangle T with vertices $(-1, 2), (-2, 5), (-5, 1)$.

(a) Translate the triangle 7 units to the right. $\Rightarrow$ ADD 7 TO X

$(-1+7, 2) = \boxed{(6,2)}$, $(-2+7, 5) = \boxed{(5,5)}$, $(-5+7, 1) = \boxed{(2,1)}$

(b) Rotate the triangle 90° counterclockwise around the origin. $\Rightarrow (x,y) \longmapsto (-y, x)$

$(-1,2) \longmapsto \boxed{(-2,-1)}$ $(-2,5) \longmapsto \boxed{(-5,-2)}$ $(-5,1) \longmapsto \boxed{(-1,-5)}$.

(c) Reflect the triangle across the line $y = x$. $\Rightarrow (x,y) \longmapsto (y,x)$.

$(-1,2) \longmapsto \boxed{(2,-1)}$
$(-2,5) \longmapsto \boxed{(5,-2)}$
$(-5,1) \longmapsto \boxed{(1,-5)}$

25. Find the distance between the points $A = (4, 8)$ and $B = (-1, -4)$. Find the coordinates of the midpoint of the line segment AB.

$d = \sqrt{(-1-4)^2 + (-4-8)^2} = \sqrt{(-5)^2 + (-12)^2} = \sqrt{25+144} = \sqrt{169} = \boxed{13}$

$M = \left(\frac{4 + -1}{2}, \frac{8 + -4}{2} \right) = \left(\frac{3}{2}, \frac{4}{2} \right) = \boxed{\left(\frac{3}{2}, 2 \right)}$

26. A line contains the points $(3, 2)$ and $(4, 5)$. Draw a line parallel to the given line that passes through the point $(0, 5)$. Draw a line perpendicular to the given line that passes through the point $(-4, 2)$.

$m_1 = \frac{5-2}{4-3} = \frac{3}{1} = 3$. POINT SLOPE: $\boxed{y - 5 = 3(x-0)}$ (PARALLEL)

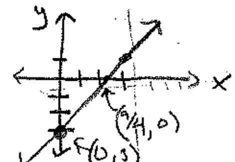
$m_2 = -\frac{1}{m_1} = -\frac{1}{3}$ POINT SLOPE: $\boxed{y - 2 = -\frac{1}{3}(x+4)}$ (PERPENDICULAR).

27. Consider the equation $-3y + 4x = 9$. Find the slope, x-intercept, and y-intercept. Sketch the graph of the equation. Does this line meet the line $-3y + 4x = 12$?

$-3y + 4x = 9 \Rightarrow -3y = -4x + 9 \Rightarrow y = \frac{4}{3}x - 3$

$\boxed{\text{Slope} = m = 4/3}$, x-int: $-3(0) + 4x = 9 \Rightarrow x = 9/4 \Rightarrow \boxed{(9/4, 0)}$

y-int: $y = 4/3(0) - 3 = -3 \longrightarrow \boxed{(0, -3)}$



$-3y = -4x + 12$

$y = \frac{4}{3}x - 4$

$\boxed{\text{NO}}$, BECAUSE BOTH HAVE THE SAME SLOPE W/ DIFFEREN y-int, SO THEY ARE PARALLEL AND SO THEY DON'T INTERSECT

28. Determine whether each of the following relations is a function. If it is a function, determine if it is possible for the function to be linear.

(a) $(9, 7), (8, 5), (7, 3), (9, 1)$

$\boxed{\text{NOT A FUNCTION}}$ BECAUSE 9 HAS TWO OUTPUTS, 7 & 1.

(b) All pairs (letter, number of lines of symmetry of the letter).

$\boxed{\text{YES}}$ - BECAUSE EVERY LETTER ALWAYS HAS THE SAME # OF LINES OF SYMMETRY.

$\boxed{\text{NOT LINEAR}}$

(c) The values of the table below:

Input, x	1	2	3	4	5
Output, y	5	8	11	14	17

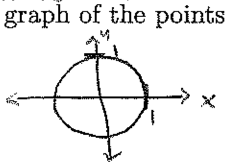
$\boxed{\text{YES}}$ IT IS A FUNCTION BECAUSE EVERY INPUT HAS ONLY 1 OUTPUT.

LINEAR BECAUSE SLOPE BETWEEN POINTS IS THE SAME FOR ALL POINTS (3).

(d) $(0, 1), (2, 5), (-4, -7), (10, 21)$

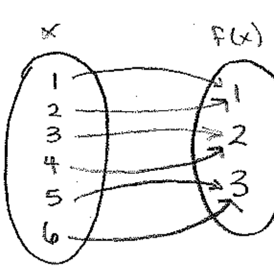
$\boxed{\text{YES}}$ IT IS A FUNCTION BECAUSE EVERY INPUT 0, 2, −4, 10 HAS ONLY 1 OUTPUT. NOTE THE SLOPE BETWEEN EACH SET OF POINTS IS THE SAME (2), SO IT CAN BE LINEAR.

(e) The graph of the points on a circle of radius 1 centered at the origin.



$\boxed{\text{NO}}$ - NOT A FUNCTION BECAUSE EACH INPUT X BETWEEN −1 AND 1 HAS TWO OUTPUTS. (ABOVE AND BELOW THE X-AXIS).

29. Draw the mapping diagram of the following relation and determine if it is a function.



$(1, 1), (2, 1), (3, 2), (4, 2), (5, 3), (6, 3)$.

$\boxed{\text{YES}}$ - IT IS A FUNCTION BECAUSE EVERY INPUT HAS EXACTLY ONE OUTPUT.