MA 114 Worksheet #22: Polar coordinates, Calculus in Polar Coordinates

1. Find the equation in polar coordinates of the line through the origin with slope $\frac{1}{3}$.
2. Find the polar equation for:
   (a) $x^2 + y^2 = 9$
   (b) $x = 4$
   (c) $y = 4$
   (d) $xy = 4$
3. Convert the equation of the circle $r = 2\sin\theta$ to rectangular coordinates and find the center and radius of the circle.
4. Find dy/dx for the following polar curves.
   (a) $r = 2\cos\theta + 1$ (b) $r = 1/\theta$
5. Compute the slope of the tangent line to the graph of $r = \sin\theta$ at $\theta = \pi/3$, and sketch the curve and the tangent line.
6. (a) Give the formula for the area of region bounded by the polar curve $r = f(\theta)$ from $\theta = a$ to $\theta = b$. Give a geometric explanation of this formula.
   (b) Give the formula for the length of the polar curve $r = f(\theta)$ from $\theta = a$ to $\theta = b$.
   (c) Use these formulas to establish the formulas for the area and circumference of a circle.
7. Find the slope of the tangent line to the polar curve $r = \theta^2$ at $\theta = \pi$.
8. Find the area enclosed by one leaf of the curve $r = \sin 2\theta$.
9. Find the arc length of one leaf of the curve $r = \sin 2\theta$.
10. Find the area in the first quadrant that lies inside the curve $r = 2\cos\theta$ and outside the curve $r = 1$.
11. Consider the sequence of circles, C_n, defined by the equations $x^2 + \left(y + \frac{1}{\sqrt{n}}\right)^2 = \frac{1}{n}$. Define a_n as the area of circle C_n and b_n as the area between circles C_n and C_{n+1}.
   (a) Sketch the picture of this infinite sequence of circles.
   (b) Does $\sum_{n=1}^{\infty} a_n$ converge?
   (c) Does $\sum_{n=1}^{\infty} b_n$ converge?
   (d) Define the circles D_n by the equations $x^2 + \left(y + \frac{1}{n}\right)^2 = \frac{1}{n^2}$ with d_n as the area of D_n. Does $\sum_{n=1}^{\infty} d_n$ converge?