MA 114 Worksheet #21: Calculus with Parametric Curves, Polar Coordinates

1. For the following parametric curves, find an equation for the tangent to the curve at the specified value of the parameter.
   (a) $x = e^{\sqrt{t}}$, $y = t - \ln(t^2)$ at $t = 1$.
   (b) $x = \cos(\theta) + \sin(2\theta)$, $y = \cos(\theta)$, at $\theta = \pi/2$.

2. For the following parametric curve, find dy/dx.
   (a) $x = e^{\sqrt{t}}$, $y = t + e^{-t}$.
   (b) $x = 4\cos(t)$, $y = \sin(2t)$.

3. Find d^2y/dx^2 for the curve $x = 7 + t^2 + e^t$, $y = \cos(t) + \dfrac{1}{t}$, $0 < t \leq \pi$.

4. Find the arc length of the following curves.
   (a) $x = 1 + 3t^2$, $y = 4 + 2t^3$, $0 \leq t \leq 1$.
   (b) $x = 4\cos(t)$, $y = 4\sin(t)$, $0 \leq t \leq 2\pi$.

5. What is the speed of the curve $c(t) = (x(t), y(t))$? Use this to find the minimum speed of a particle with trajectory $c(t) = (t^2, 2\ln(t))$, for $t > 0$.

6. Suppose you wrap a string around a circle. If you unwind the string from the circle while holding it taut, the end of the string traces out a curve called the *involute* of the circle. Suppose you have a circle of radius r centered at the origin, with the end of the string all the way wrapped up resting at the point $(r, 0)$. As you unwrap the string, define θ to be the angle formed by the x-axis and the line segment from the center of the circle to the point up to which you have unwrapped the string.
   (a) Draw a picture and label θ.
   (b) Show that the parametric equations of the involute are given by $x = r(\cos\theta + \theta\sin\theta)$, $y = r(\sin\theta - \theta\cos\theta)$.
   (c) Find the length of the involute for $0 \leq \theta \leq 2\pi$.

7. Convert from rectangular to polar coordinates:
   (a) $(1, \sqrt{3})$
   (b) $(-1, 0)$

8. Convert from polar to rectangular coordinates:
   (a) $\left(2, \dfrac{\pi}{6}\right)$
   (b) $\left(-1, \dfrac{\pi}{2}\right)$

9. Sketch the graph of the polar curves:

   (a) $\theta = \dfrac{3\pi}{4}$

   (b) $r = \pi$

   (c) $r = \cos\theta$

   (d) $r = \cos(2\theta)$

   (e) $r = 1 + \cos\theta$