MA 114 Worksheet #13: Power Series

1. (a) Define the terms *power series*, *radius of convergence*, and *interval of convergence.*

   (b) Give the definition of the radius of convergence of a power series $\sum_{n=0}^{\infty} a_n x^n$

   (c) For what values of x does the series $\sum_{n=1}^{\infty} 2\big(\cos(x)\big)^{n-1}$ converge?

   (d) Find a formula for the coefficients c_k of the power series $\frac{1}{0!}+\frac{2}{1!}x+\frac{3}{2!}x^2+\frac{4}{3!}x^3+\cdots$.

   (e) Find a formula for the coefficients c_n of the power series $1+2x+x^2+2x^3+x^4+2x^5+x^6+\cdots$.

   (f) Suppose $\lim_{n\to\infty} \sqrt[n]{|c_n|} = c$ where $c \neq 0$. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$.

   (g) Consider the function $f(x) = \dfrac{5}{1-x}$. Find a power series that is equal to $f(x)$ for every x satisfying $|x| < 1$.

2. Find the radius and interval of convergence for

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n n}{4^n}(x-3)^n$.

   (b) $\displaystyle 4\sum_{n=0}^{\infty} \frac{2^n}{n}(4x-8)^n$.

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{(-3)^n}$.

   (d) $\displaystyle\sum_{n=0}^{\infty} n!(x-2)^n$.

   (e) $\displaystyle\sum_{n=0}^{\infty} (5x)^n$

   (f) $\displaystyle\sum_{n=0}^{\infty} \sqrt{n}x^n$

   (g) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n}}$

   (h) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{3^n \ln n}$

   (i) $\displaystyle\sum_{n=0}^{\infty} \frac{(x=2)^n}{n^n}$

   (j) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n^4}$

   (k) $\displaystyle\sum_{n=0}^{\infty} \frac{(5x)^n}{n^3}$

3. Use term by term integration and the fact that $\displaystyle\int \frac{1}{1+x^2}\,dx = \arctan(x)$ to derive a power series centered at $x=0$ for the arctangent function. Hint: $\dfrac{1}{1+x^2} = \dfrac{1}{1-(-x^2)}$.

4. Use the same idea as above to give a series expression for $\ln(1+x)$, given that $\dfrac{dx}{1+x} = \ln(1+x)$. You will again want to manipulate the fraction $\dfrac{1}{1+x} = \dfrac{1}{1-(-x)}$ as above.

5. Write $(1+x^2)^{-2}$ as a power series. Hint: use term by term differentiation.