

MA 114 Worksheet #03: Special Trigonometric Integrals

1. Compute the following integrals:

(a) $\int_0^2 \frac{u^3}{\sqrt{16-u^2}} du$

(b) $\int \frac{1}{x^2 \sqrt{25-x^2}} dx$

(c) $\int_0^{\pi/2} \cos^2(x) dx$

(d) $\int \sqrt{\cos x} \sin^3 x dx$

(e) $\int_0^{2\pi} \sin^2\left(\frac{1}{3}\theta\right) d\theta$

(f) $\int_0^{\pi/2} (2 - \sin \theta)^2 d\theta$

(g) $\int \frac{\sqrt{1-x^2}}{x^4} dx$

(h) $\int_0^3 \frac{x}{\sqrt{36-x^2}} dx$. Hint: Use the substitution $x = 6u$.

(i) $\int_0^{1/2} x\sqrt{1-4x^2} dx$. Hint: Substitute $x = u/2$.

2. Let $r > 0$. Consider the identity

$$\int_0^s \sqrt{r^2 - x^2} dx = \frac{1}{2}r^2 \arcsin(s/r) + \frac{1}{2}s\sqrt{r^2 - s^2}$$

where $0 \leq s \leq r$.

(a) Plot the curves $y = \sqrt{r^2 - x^2}$, $x = s$, and $y = \frac{x}{s}\sqrt{r^2 - x^2}$.

(b) Using part (a), verify the identity geometrically.

(c) Verify the identity using trigonometric substitution.