

Worksheet # 4: Review of Trigonometry (continued)

1. Find the exact values of the following expressions. Do not use a calculator.

- (a) $\cos(\sin^{-1}(0.28))$
- (b) $\tan^{-1}(\tan(31\pi/4))$
- (c) $\cos(2 \sin^{-1}(-\frac{2}{3}))$
- (d) $\tan(2 \tan^{-1}(\frac{1}{3}))$
- (e) $\sin(\cos^{-1}(\frac{4}{5}) + \sin^{-1}(\frac{12}{13}))$

2. Show the following:

- (a) $\tan(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}$
- (b) $\cos(2 \sin^{-1} x) = 1 - 2x^2$

3. Find all solutions to the following equations in the interval $[0, 2\pi]$.

- (a) $\sin \theta + 1 = \cos \theta$
- (b) $3 \sec^2 \theta - 2 \tan^2 \theta - 4 = 0$
- (c) $2 \sin(2\theta) = -\sqrt{3}$

4. Which of the following are odd functions? Even functions? Neither?

- (a) $x \sin x$
- (b) $\sin^2 x$
- (c) $\csc x$
- (d) $|\sin x|$
- (e) $\sin(\cos x)$
- (f) $x + \sin x$

5. Use the tangent addition formula

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

to verify that

$$\frac{\pi}{4} = 4 \tan^{-1} \left(\frac{1}{5} \right) - \tan^{-1} \left(\frac{1}{239} \right).$$

This result was discovered by John Machin in 1706 and used by him to calculate the first 100 decimal places of π (without a computer!). [*Hint: start out by setting $x = y = \tan^{-1}(1/5)$ in the tangent addition formula.*]