Related Rates

- Given multiple variables, find ROC of one in terms of the others

<u>Ex1</u>: Air is pumped into a spherical balloon at a rate $50\ cm^3/s$.

How fast does the radius change when the radius is 20 cm?

<u>Variables</u>: V (volume), r (radius) t (time)

<u>Goal</u>: $\frac{dr}{dt}$ when $r=20$ cm

Know: $V = \frac{4}{3}\pi r^3$, $\frac{dV}{dt} = 50 \text{ cm}^3/\text{s}$

Differentiate: $\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

Solve: $\frac{dr}{dt} = \frac{1}{4\pi r^2} \cdot \frac{dV}{dt}$

$$\frac{1}{\text{cm}^2} \cdot \text{cm}^3/\text{s} = \text{cm}/\text{s}$$

$$\left.\frac{dr}{dt}\right|_{r=20} = \frac{1}{4\pi(20)^2} \cdot 50 \text{ cm/s}$$

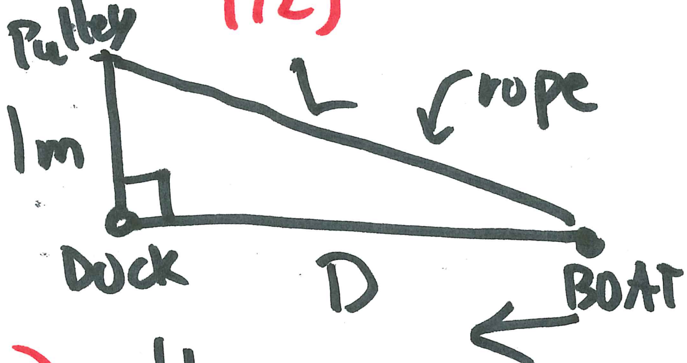
$$= \boxed{\frac{50}{1600\pi} \text{ cm/s}}$$

General Strategy

1) Identify variables
2) Draw picture
3) What do you know?
4) What do you want?
5) How are the variables related?
   - write down equations
6) Differentiate (use chain rule)
7) Evaluate.

Ex 2: Boat pulled to a dock by a rope. Rope is passing through a pulley 1m higher than the boat. If the rope is pulled at a rate of 1 m/s, how fast is the boat approaching when it is 8m from the dock?

1/2)



D = dist. from the dock

L = length of rope

t = time

3) $\frac{dL}{dt} = -1 \text{ m/s}$ 4) $\left.\frac{dD}{dt}\right|_{D=8m}$

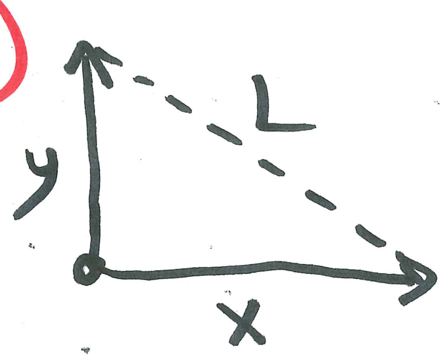
5) Pyth. Thm: $1^2 + D^2 = L^2$

$$\boxed{\begin{array}{l} \boxed{D=8} \\ 1+64 = L^2 \\ L = \sqrt{65} \end{array}}$$

6) $0 + 2D \cdot \frac{dD}{dt} = 2L \cdot \frac{dL}{dt}$

7) $\frac{dD}{dt} = \frac{L}{D} \cdot \frac{dL}{dt} = \frac{\sqrt{65}}{8}(-1) = \boxed{-\frac{\sqrt{65}}{8} \text{ m/s}}$

Ex 3: Two people walk from the same starting point, one east at 2 mi/hr, one north at 3 mi/hr. How fast is the distance between them changing after one hour?

1/2) 

x = Dist. of 1st person from start
y = Dist of 2nd person from start
L = Dist. between them
t = time

3) $\frac{dx}{dt} = 2\text{mph}$ $\quad \frac{dy}{dt} = 3\text{mph}$

4) $\left.\frac{dL}{dt}\right|_{t=1}$ $\quad$ 5) $x^2 + y^2 = L^2$

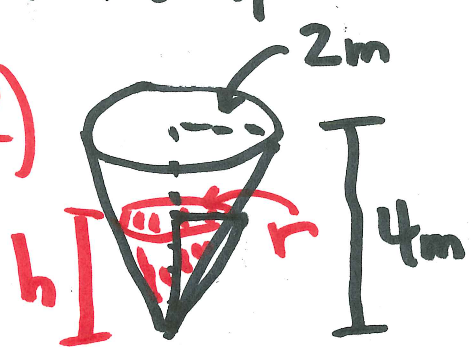
6) Take $\frac{d}{dt}$: $\cancel{2}x \cdot \frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}L\frac{dL}{dt}$

$$\frac{dL}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{L} = \frac{2x+3y}{\sqrt{x^2+y^2}}$$

7) $\left.\frac{dL}{dt}\right|_{t=1} = \frac{2\cdot 2 + 3\cdot 3}{\sqrt{2^2+3^2}} = \frac{13}{\sqrt{13}} = \boxed{\sqrt{13}\text{ mph}}$

Ex: A water tank is an inverted circular cone w/ base radius 2m & height 4m. If water is pumped in at a rate of 2 m³/min, find the rate at which the water level is rising when the water is 3m deep.

1/2)



r = radius of cone of water
h = height of cone of water
V = volume of cone of water
t = time

3) $\frac{dV}{dt} = 2 \text{ m}^3/\text{min}$ 4) $\left.\frac{dh}{dt}\right|_{h=3m}$

5) • similar Δs ${}_h\nabla^{r} \sim {}_4\nabla^{2} \Rightarrow \frac{r}{2} = \frac{h}{4}$

$r = \frac{h}{2}$

• $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$= \frac{1}{12}\pi h^3$

6) $\frac{dV}{dt} = \frac{1}{4}\pi h^2 \cdot \frac{dh}{dt}$ $\quad \frac{1}{12} \cdot 3 = \frac{1}{4}$

$\frac{dh}{dt} = \frac{4}{\pi h^2} \cdot \frac{dV}{dt}$

7) $\left.\frac{dh}{dt}\right|_{h=3} = \frac{4}{9\pi} \cdot 2 = \boxed{\frac{8}{9\pi} \text{ m/min}}$