MathExcel Supplemental Problems E: Using Derivative Rules

1. Find the following derivatives and simplify your answer. $y^{(n)} = f^{(n)}(x)$ denotes the n^{th} derivative of $y = f(x)$.

   (a) $y^{(4)}$, $y = e^x \cos x$.

   (b) $g''(0)$, $g(s) = \dfrac{e^s}{s+1}$.

   (c) y', $y = \dfrac{\sin x - x\cos x}{\cos x + x \sin x}$.

   (d) y', $y = \tan x - \cot x$

   (e) $f'(x)$. $f(x) = \left(1 + \cot^5(x^4+1)\right)^9$.

   (f) $h'(x)$, $h(x) = e^{e^{e^x}}$.

2. The power, P, that a battery supplies to a device depends on the internal resistance of the battery. For a battery of voltage V and an internal resistance r, the total power delivered to a device with resistance R is give by the formula

$$P = \frac{V^2 R}{(R+r)^2}.$$

   Assuming that V and R are constants, determine $\dfrac{dP}{dr}$.

3. If $y = f(u)$ and $u = g(x)$, where f and g are twice differentiable functions, show that

$$\frac{d^2y}{dx^2} = \frac{d^2y}{du^2}\left(\frac{du}{dx}\right)^2 + \frac{dy}{du}\frac{d^2u}{dx^2}$$

4. (a) Write $|x| = \sqrt{x^2}$ and use the chain rule to show that

$$\frac{d}{dx}|x| = \frac{x}{|x|}$$

   (b) If $f(x) = |\sin x|$, find $f'(x)$ and sketch the graphs of f and f'. Where is f not differentiable?

   (c) If $g(x) = \sin|x|$, find $g'(x)$ and sketch the graphs of g and g'. Where is g not differentiable?

5. Consider the parabola $y = x^2 + 1$. How many different tangent lines to this graph would cross through a point (a, b) given the following conditions on a and b? Draw the graph and the tangent line(s) for each case.

   (a) $b < a^2 + 1$

   (b) $b = a^2 + 1$

   (c) $b > a^2 + 1$

6. Find all values of n and x such that $y = x^n$ satisfies $x^2y'' - 2xy' = 4y$.

7. If f is a differentiable function that satisfies $\cos(f(x)) = x$, use the chain rule to show that $f'(x) = \dfrac{-1}{\sqrt{1-x^2}}$. (Hint: take the derivative of both sides of the first equation with respect to x. It may also be helpful to draw a certain triangle.)

8. Find a quadratic function $p(x)$ such that $p(2) = 3$, $p'(1) = 14$, and $p''(10) = 4$.

9. At time t seconds, the center of a bobbing cork is $3\sin 2t$ centimeters above (or below) water level. What is the velocity of the cork at $t = 0, \frac{\pi}{2}, \pi$?

10. Find constants A and B such that the function $y = A\sin x + B\cos x$ satisfies the differential equation $y'' + y' - 2y = \sin x$.

11. Assume that f, g and w are differentiable functions. It can be shown that if $h(x) = f(x)g(x)w(x)$, then by the product rule we have

$$h'(x) = f'(x)g(x)w(x) + f(x)g'(x)w(x) + f(x)g(x)w'(x)$$

(a) Use the formula above to show that

$$\frac{d}{dx}[f(x)]^3 = 3[f(x)]^2 f'(x).$$

(b) Use part (a) to differentiate $y = e^{3x}$.