MathExcel Supplemental Worksheet B: Functions, average velocities, and limits

1. Find an expression for a function whose graph consists of a line segment joining the point $(-2, 2)$ to $(-1, 0)$ together with the top half of the unit circle with center at the origin.

2. Rebecca sets out on a journey. For the first half of the distance, she drives leisurely at 30 miles/ hour and for the second half of the distance, she drives at 60 miles/hour. What is her average speed?

3. Consider an object moving with a position given by the function $f(t) = t^2$ and the point $P(1, 1)$ on the graph of $f(t)$.

   (a) Compute the average velocity of the object between P and each point Q_i for i from 1 to 9:
   $Q_1 = (2, f(2)), Q_2 = (1.5, f(1.5)), Q_3 = (1.1, f(1.1)), Q_4 = (1.01, f(1.01)),$
   $Q_5 = (1.001, f(1.001)), Q_6 = (0, f(0)), Q_7 = (0.9, f(0.9)), Q_8 = (0.99, f(0.99)),$
   $Q_9 = (0.999, f(0.999))$

   (b) Using the above data, estimate the instantaneous velocity of the object at time $t = 1$.

4. Decide whether the following statements are true always/sometimes/never. Justify your answer in each case.

   (a) As x approaches 100, the function $f(x) = \frac{1}{x}$ gets closer and closer to 0, so the limit as x goes to 100 of $f(x)$ is 0.

   (b) $\lim_{x \to a} f(x) = L$ means that if x_1 is closer to a than x_2, then $f(x_1)$ will be closer to L than $f(x_2)$ is.

   (c) Whether of not $\lim_{x \to a} f(x) = L$ exists, depends on how $f(a)$ is defined.

   (d) If $f(x) = \frac{x^2-4}{x-2}$ and $g(x) = x + 2$, then we can say that f and g are equal.

   (e) You are trying to guess $\lim_{x \to 0} f(x)$. You plug in $x = 0.1, 0.01, 0.001, \ldots$ and get $f(x) = 0$ at all those values. In fact, you are told that for all $n = 1, 2, \ldots,$ $f(\frac{1}{10^n}) = 0$. Then, we can conclude that $\lim_{x \to 0} f(x) = 0$

5. Consider the following function

$$f(x) = \begin{cases} x^2 & x \text{ is rational}, x \neq 0 \\ -x^2 & x \text{ is irrational} \\ \text{undefined} & x = 0. \end{cases} \tag{1}$$

Determine which of the following statements is true.

(a) There is no a for which $\lim_{x\to a} f(x)$ exists.

(b) There may be some a for which $\lim_{x\to a} f(x)$ exists, but it is impossible to say without more information.

(c) $\lim_{x\to a} f(x)$ exists only if $a = 0$.

(d) $\lim_{x\to a} f(x)$ exists for infinitely many a.

6. Sketch the graph of an example of a function f that satiesfies the given conditions.

(a) $\lim_{x\to 2^-} f(x) = 1$, $\lim_{x\to 1^+} f(x) = 1$, $f(0) = 1$

(b) $\lim_{x\to 0} f(x) = 1$, $\lim_{x\to 1^-} f(x) = 0$, $\lim_{x\to 1^+} f(x) = -1$, $f(1) = 1$

(c) $\lim_{x\to 3^-} f(x) = \infty$, $\lim_{x\to 3^+} f(x) = -\infty$, $\lim_{x\to 2} f(x) = \infty$, $\lim_{x\to 4} f(x) = -\infty$

7. Carefully use the limit laws and the fact that $\lim_{x\to c} x^n = c^n$ to evaluate the following limits. Show all your work.

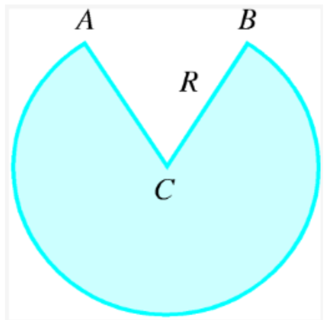
(a) $\lim_{t\to 4} \dfrac{3t - 14}{t + 1}$

(b) $\lim_{z\to 9} \dfrac{\sqrt{z}}{z - 2}$

(c) $\lim_{y\to \frac{1}{3}} (18y^2 - 4)^4$

(d) $\lim_{t\to 0} \dfrac{t^2 + 1}{(t^3 + 2)(t^4 + 1)}$

8. (Review) A cone shaped drinking cup is made from a circular piece of paper of radius R by cutting out a sector and joining the edges CA and CB. Let r and h denote the base radius and height of the conical cup, respectively. Express the volume of the conical cup as function of h.



(*Hint:* For a right circular cone with base radius r, height h and slant height l, $(l)^2 = (r)^2 + (h)^2$)

9. (Review) The half life of Palladium-100 (^{100}Pd) is 4 days. Suppose you start with an initial sample of 1 gram, then

   (a) Find the mass of ^{100}Pd that is left after 16 days.

   (b) Let $m(t)$ denote the mass of ^{100}Pd left at t days. Express $m(t)$ as a function of t.

   (c) Find the inverse of $m(t)$ and explain its meaning.

   (d) When will the mass of ^{100}Pd be reduced to 0.01 grams?

10. (Review) Consider the function $f_0(x) = \frac{x}{x+1}$

    (a) Compute the following compositions

       1. $f_1(x) = f_0 \circ f_0$
       2. $f_2(x) = f_0 \circ f_1$
       3. $f_3(x) = f_0 \circ f_2$

    (b) Do you notice a pattern? Can you guess the expression for the function $f_n(x)$ for any $n \geq 0$?

    (c) Graph $f_0(x), f_1(x), f_2(x)$ and $f_3(x)$ on the same screen and describe the effects of repeated composition.