MATH 2551-G Exam 1

Fall 2025

Full name: _____ **GT ID:**_____

Read all instructions carefully before beginning.

- Print your name and GT ID neatly above.
- You have 75 minutes to complete the problems.
- You may not use electronic devices of any kind during the exam. You may not use any reference materials other than your single page of hand-written notes you brought to the exam.
- The Learning Targets covered by this exam are listed below.
- Show your work. Answers without work shown will receive a **Not Yet**
- Good luck! Write yourself a message of encouragement on the front page!

Learning Targets

- **G1: Lines and Planes.** I can describe lines using the vector equation of a line. I can describe planes using the general equation of a plane. I can find the equations of planes using a point and a normal vector. I can find the intersections of lines and planes. I can describe the relationships of lines and planes to each other. I can solve problems with lines and planes.
- **G2: Calculus of Curves.** I can compute tangent vectors to parametric curves and their velocity, speed, and acceleration. I can find equations of tangent lines to parametric curves. I can solve initial value problems for motion on parametric curves.
- **G3: Geometry of Curves.** I can compute the arc length of a curve in two or three dimensions and apply arc length to solve problems. I can compute normal vectors and curvature for curves in two and three dimensions. I can interpret these objects geometrically and in applications.
- **G4: Surfaces.** I can identify standard quadric surfaces including: spheres, ellipsoids, elliptic paraboloids, hyperboloids, cones, and hyperbolic paraboloids. I can match graphs of functions of two variables to their equations and contour plots and determine their domains and ranges.
- **G5: Parameterization.** I can find parametric equations for common curves, such as line segments, graphs of functions of one variable, circles, and ellipses. I can match given parametric equations to Cartesian equations and graphs. I can parameterize common surfaces, such as planes, quadric surfaces, and functions of two variables.
- **D1: Computing Derivatives.** I can compute partial derivatives, total derivatives, directional derivatives, and gradients. I can use the Chain Rule for multivariable functions to compute derivatives of composite functions.

1. **[G1: Lines and Planes]** Consider the plane p with equation $2x - y + 4z = 5$.

   (a) Give a line ℓ through the point $(10, 20, 30)$ which is orthogonal to p.

2. [**G2: Calculus of Curves**]

(a) **True/False:** If $\mathbf{r}(0) = \langle 3, 2, 1\rangle$ and $\mathbf{r}'(t) = \langle x(t), y(t), z(t)\rangle$ then

$$\mathbf{r}(t) = \int \langle x(t), y(t), z(t)\rangle \, dt + \langle 3, 2, 1\rangle.$$

(b) Give an equation for the tangent line to the curve parameterized by

$$\mathbf{r}(t) = \langle e^t, 4t - 1, t^2\rangle$$

at the point $(e, 3, 1)$.

3. [**G3: Geometry of Curves**]

(a) Order the following curves in increasing order of curvature.

A) A helix of curvature $1/100$
B) A line segment of length 100
C) A circle of radius 2

(b) Find the coordinates of the point which lies a distance $2\sqrt{13}\pi/3$ along the helix

$$\mathbf{r}(t) = \langle 2\cos(t), -2\sin(t), 3t\rangle$$

in the direction of increasing parameter t from $(2, 0, 0)$.

4. **[G4: Surfaces]**

   (a) Find and sketch the domain of the function

$$f(x,y) = \sqrt{9 - x^2 - y^2} + \frac{2x}{3y - 6}.$$

   Clearly indicate any points which are not included.

5. [**G5: Parameterization**] The sphere $x^2+y^2+z^2=8$ meets the paraboloid $2z=x^2+y^2$ in a circle. Give a parameterization of this circle. Be sure to specify a domain.

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