Math 2550 Worksheet - Week 1

Learning Targets

- **G1: Lines and Planes.** I can describe lines using the vector equation of a line. I can describe planes using the general equation of a plane. I can find the equations of planes using a point and a normal vector. I can find the intersections of lines and planes. I can describe the relationships of lines and planes to each other. I can solve problems with lines and planes.
- **G4: Surfaces.** I can identify standard quadric surfaces including: spheres, ellipsoids, elliptic paraboloids, hyperboloids, cones, and hyperbolic paraboloids. I can match graphs of functions of two variables to their equations and contour plots and determine their domains and ranges.

Warmup

1. If the statement is *always* true, answer true. If the statement is *ever* false, answer false. Justify your answer.

   In each case, $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in $\mathbb{R}^3$.

   (a) $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$

   (b) $\mathbf{u} \cdot \mathbf{u} = |\mathbf{u}|^{\mathbf{2}}$

   (c) $(\mathbf{u} \times \mathbf{u}) \cdot \mathbf{u} = \mathbf{0}$

2. Suppose $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in $\mathbb{R}^3$. Which of the following make sense, and which do not? For those that make sense, is the result a vector or a scalar?

   (a) $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$

   (b) $\mathbf{u} \times (\mathbf{v} \cdot \mathbf{w})$

   (c) $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$

   (d) $\mathbf{u} \cdot (\mathbf{v} \cdot \mathbf{w})$

Focus Problems

3. **[G1]** Find an equation for

   (a) the line through point $P = (1, 2, -1)$ and point $Q = (-1, 0, 1)$.

   (b) the line through $(0, -7, 0)$ perpendicular to the plane $x + 2y + 2z = 13$.

   (c) the plane containing the point $(2, 1, 1)$ and orthogonal to the vector $\langle 2, 2, -2\rangle$

4. **[G1]** Find a vector in the direction of the line of intersection ℓ of the planes $2x+y-z = 3$ and $x + 2y + z = 2$. Find a plane which goes through $(2, 1, -1)$ and is perpendicular to ℓ (and thus both planes).

5. **[G4]** For the following, identify and describe (for example, which way is it oriented? what are the cross-sections?) the type of surface.

   (a) $x^2 + 4z^2 = 16$.

   (b) $9x^2 + z^2 + y^2 = 9$.

   (c) $y^2 = 3x^2 + 3z^2$.

   (d) $z = x^2 + y^2 + 4$.

   (e) $x^2 + y^2 = 16 - z^2$

Extra Problems

6. **[G1]** Find 2 planes that are not parallel that both contain the points $P(1, -1, 1)$, $Q(3, 2, 0)$, and $R(5, 5, -1)$. When will 3 distinct points NOT determine a unique plane?

7. **[G1]** Find the point where the line $\mathbf{r}(t) = \langle 2, 3+2t, 1+t \rangle$ intersects the plane $2x-y+3z = 6$.

8. **[G1]** How can you tell when two planes $A_1x+B_1y+C_1z = D_1$ and $A_2x+B_2y+C_2z = D_2$ are parallel? Perpendicular? Justify your answer.

9. **[G1]** Find the point at which the lines $\ell_1(t) = \langle 2, 3, 1 \rangle + \langle 1, -1, 1 \rangle t$ and $\ell_2(t) = \langle 2, 1, -2 \rangle t + \langle 6, 2, 1 \rangle$ intersect.