MATH 2551 Guided Notes

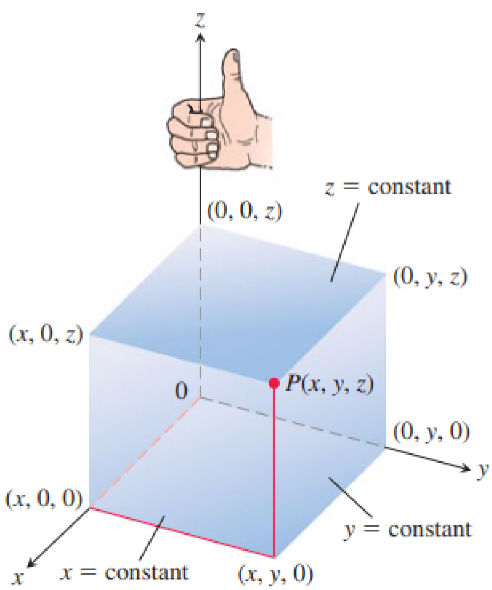
Dr. Hunter Lehmann (Dr. H, Dr. Lehmann, Dr. Hunter)

Fall 2025

Day 1 - Course Introduction and Cross Products

Pre-Lecture

Section 12.1: Three-Dimensional Coordinate Systems



Day 1 - Lecture

Daily Announcements & Reminders:

Goals for Today:

Sections 12.1, 12.3, 12.4

- Set classroom norms
- Describe the big-picture goals of the class
- Review $\mathbb{R}^3$ and the dot product
- Introduce the cross product and its properties

Icebreaker on PollEverywhere

Introduction to the Course

Purposes:

- Pre-Lecture: Get in math headspace, first exposure to the day's topic
- Lecture: Fill in the rest of the new ideas for the week
- Studio: Guided group practice with new ideas under supervision, develop independence
- WeBWorK: Basic skills practice for the topics
- LT Practice: Extra practice problems for assessments
- Quizzes, Checkpoints, & Exams: Demonstrate learning & get feedback

Feedback loops: Lecture -> practice -> studio -> practice -> assessment -> practice -> assessment -> ...

Important Canvas Items: Back to Canvas!

Big Idea: Extend differential & integral calculus.

What are some key ideas from these two courses?

<u>Differential Calculus</u> <u>Integral Calculus</u>

Poll

Before: we studied **single-variable functions** $f : \mathbb{R} \to \mathbb{R}$ like $f(x) = 2x^2 - 6$.

Now: we will study **multi-variable functions** $f : \mathbb{R}^n \to \mathbb{R}^m$: each of these functions is a rule that assigns one output vector with m entries to each input vector with n entries.

Example 1. What shape is the set of solutions $(x, y, z) \in \mathbb{R}^3$ to the equation

$$x^2 + y^2 = 1?$$

Section 12.3/4: Dot & Cross Products

Definition 2. The **dot product** of two vectors $\mathbf{u} = \langle u_1, u_2, \dots, u_n \rangle$ and $\mathbf{v} = \langle v_1, v_2, \dots, v_n \rangle$ is

$$\mathbf{u} \cdot \mathbf{v} = \underline{\hspace{8cm}}$$

This product tells us about ____.

In particular, two vectors are **orthogonal** if and only if their dot product is ______.

Example 3. Are $\mathbf{u} = \langle 1, 1, 4 \rangle$ and $\mathbf{v} = \langle -3, -1, 1 \rangle$ orthogonal?

Goal: Given two vectors, produce a vector orthogonal to both of them in a "nice" way.

1.

2.

Definition 4. The **cross product** of two vectors $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ <u>in $\mathbb{R}^3$</u> is

$$\mathbf{u} \times \mathbf{v} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Example 5. Find $\langle 1, 2, 1\rangle \times \langle 3, -1, 0\rangle$.

Day 2 - Lines, Planes, and Quadrics

Pre-Lecture

12.5: Lines

Lines in $\mathbb{R}^2$, a new perspective:

Example 6. Find a vector equation for the line that goes through the points $P = (1, 0, 2)$ and $Q = (-2, 1, 1)$.

Day 2 Lecture

Daily Announcements & Reminders:

Learning Targets:

- **G1: Lines and Planes.** I can describe lines using the vector equation of a line. I can describe planes using the general equation of a plane. I can find the equations of planes using a point and a normal vector. I can find the intersections of lines and planes. I can describe the relationships of lines and planes to each other. I can solve problems with lines and planes.
- **G4: Surfaces.** I can identify standard quadric surfaces including: spheres, ellipsoids, elliptic paraboloids, hyperboloids, cones, and hyperbolic paraboloids. I can match graphs of functions of two variables to their equations and contour plots and determine their domains and ranges.

Goals for Today:

Sections 12.5-12.6

- Apply the cross product to solve problems
- Learn the equations that describe lines, planes, and quadric surfaces in $\mathbb{R}^3$
- Solve problems involving the equations of lines and planes
- Sketch quadric surfaces in $\mathbb{R}^3$

Example 7. Find a set of parametric equations for the line through the point $(1, 10, 100)$ which is parallel to the line with vector equation

$$\mathbf{r}(t) = \langle 1, 4, -3 \rangle t + \langle 0, -1, 1 \rangle$$

Section 12.5 Planes

Planes in $\mathbb{R}^3$

Conceptually: A plane is determined by either three points in $\mathbb{R}^3$ or by a single point and a direction $\mathbf{n}$, called the *normal vector*.

Algebraically: A plane in $\mathbb{R}^3$ has a *linear* equation (back to Linear Algebra! imposing a single restriction on a 3D space leaves a 2D linear space, i.e. a plane)

In $\mathbb{R}^3$, a pair of lines can be related in three ways:

parallel **skew** **intersecting**

On the other hand, a pair of planes can be related in just two ways:

parallel **intersecting**

Example 8. [Poll] The lines

$$\ell_1(t) = \langle 1, 1, 1\rangle t + \langle 0, 0, 1\rangle$$

and

$$\ell_2(t) = \langle 2, 2, 2\rangle t + \langle 0, 0, 1\rangle$$

are related in what way?

Example 9. [Poll] The lines

$$\ell_1(t) = \langle 1, 1, 1\rangle t + \langle 0, 0, 1\rangle$$

and

$$\ell_2(t) = 2t\mathbf{i} - t\mathbf{j} - t\mathbf{k}$$

are related in what way?

Example 10. Consider the planes $y - z = -2$ and $x - y = 0$. Show that the planes intersect and find an equation for the line of intersection of the planes.